In this worksheet you will master the process of solving equations and inequalities that involve absolute value expressions. You will work through a range of problems from straightforward equations to more challenging inequalities and proofs.

Easy Questions

1. Solve $|x| = 5$.
2. Solve $|x - 3| = 0$.
3. Solve $|x + 2| = 7$.
4. Solve $|2x| = 10$.
5. Solve $|x - 1| = 1$.

Intermediate Questions

6. Solve $|x - 3| = 4$.
7. Solve $|2x + 1| = 3$.
8. Solve $|x + 5| = 2$.
9. Solve $|4 - x| = 3$.
10. Solve $|3x| = 9$.
11. Solve $|x + 3| = |x - 1|$.
12. Solve $|x - 2| < 5$.
13. Solve $|2x + 3| \leq 7$.
14. Solve $|x + 1| > 3$.
15. Solve $|3x - 2| \geq 4$.
16. Solve $|x - 1| < 3$ and draw the number line representation on your paper.
17. Express the solution set of $|x + 4| \leq 8$ in interval notation.
18. Solve $|2x - 5| < 7$ and simplify your answer.
19. Solve $|x| \geq 2$.
20. Explain how the inequality $|x - a| < b$ represents the concept of distance.

Hard Questions

21. Solve $||x-2|-3|=1$.
22. Solve $|x-3|+|x+1|<7$.
23. Solve $|2x-|x-1||=3$.
24. Prove that $|x+2|-|x-2| \leq 2$ for all x. Write a short explanation.
25. Solve $||x+1|-2|>1$ and express the solution in set builder notation.
26. Solve $|2(x-3)|=8$ and explain each step.
27. Solve $|3x+4| \geq 10$ and represent the solution on a number line.
28. Solve $|x-5| \leq |x+1|$ and discuss the critical points.
29. Determine all values of x that satisfy $|x-2|+|x+2|=6$.
30. For what values of a does the equation $|x-a|=|x+a|$ have all real numbers as solutions? Give a reason.