ILLAWARRA TUTORING

In this worksheet you will determine the variance and standard deviation of a distribution in order to measure how spread out the outcomes are. You will practice calculating the mean, variance and standard deviation using discrete probability distributions along with explaining the concepts behind these measures.

Easy Questions

1. Write in your own words the definitions of σ^2 (variance) and σ (standard deviation). Explain how they measure the spread of a distribution.

2. Consider a discrete probability distribution with outcomes 1 and 3 and probabilities 0.5 each. Compute the expected value μ, the variance σ^2 using the definition $\sigma^2 = \sum[p(x)(x-\mu)^2]$ and then determine the standard deviation σ.

3. Using the formula $\sigma^2 = \sum[p(x)(x-\mu)^2]$, calculate the squared deviation for an outcome of 4 when the mean is 3 and the probability is 0.2.

4. If a distribution has a variance of 9, what is its standard deviation? Show your work.

5. A random variable takes the values 2 and 8 each with a probability of 0.5. Compute the expected value, and hence find the variance and standard deviation.

Intermediate Questions

6. Consider a discrete distribution with outcomes 1, 2 and 3, and corresponding probabilities 0.3, 0.4 and 0.3. Calculate the expected value μ and then use it to compute the variance σ^2 and standard deviation σ.

7. For a random variable with outcomes 2, 5 and 7 with probabilities 0.2, 0.5 and 0.3 respectively, first compute $E(x)$ and $E(x^2)$. Then use the formula $\sigma^2 = E(x^2) - [E(x)]^2$ to find the variance, and deduce the standard deviation.

8. A discrete random variable takes the values -1, 0 and 1 with probabilities 0.3, 0.4 and 0.3 respectively. Determine the mean and then calculate the variance and standard deviation.

9. Given a distribution with outcomes 4, 6 and 8 and probabilities 0.2, 0.5 and 0.3 respectively, compute the expected value, variance and standard deviation.

10. Explain why the variance σ^2 is always non-negative. Support your explanation by referring to the formula $\sigma^2 = \sum[p(x)(x-\mu)^2]$.

11. A discrete distribution has outcomes 1, 2 and 3 with probabilities 0.2, p and 0.5 respectively. Determine the missing probability p and then calculate the expected value and variance.

12. Suppose a random variable always takes the value 5. Show that its expected value is 5, and prove that its variance and standard deviation are both zero.

13. A random variable takes the values 10 and 20 with probabilities 0.7 and 0.3 respectively. Calculate the expected value, variance and standard deviation.

14. Compute the variance and standard deviation for a random variable with outcomes 2, 3 and 5 with probabilities 0.25, 0.5 and 0.25 respectively. Use the method of summing the squared deviations.

15. Consider the outcomes 0, 1, 2 and 3 each with equal probability. First determine the expected value and then compute the variance and standard deviation.

16. A random variable takes the values 3, 5 and 8 with probabilities 0.2, 0.5 and 0.3 respectively. Find the expected value, variance and standard deviation.

17. Calculate the variance and standard deviation for a discrete distribution with outcomes -2, 0 and 2 and probabilities 0.25, 0.5 and 0.25 respectively.

18. For a random variable taking values 1, 4 and 7 with probabilities 0.3, 0.4 and 0.3 respectively, compute the mean, variance and standard deviation.

19. Provide a derivation of the variance formula starting from $\sigma^2 = \sum[p(x)(x-\mu)^2]$. Explain each step and why the formula is equivalent to $\sigma^2 = E(x^2) - [E(x)]^2$.

20. A random variable takes the values 10, 15 and 20 with probabilities 0.2, 0.5 and 0.3 respectively. Calculate the expected value, variance and standard deviation. Provide a brief interpretation of what the standard deviation indicates about the spread of the distribution.

Hard Questions

21. A teacher records the scores of a test and notes the following discrete distribution for marks: outcomes 70, 80, 90 and 100 with probabilities 0.1, 0.3, 0.4 and 0.2 respectively. Compute the expected score, variance and standard deviation. Explain how these measures can help in understanding the spread of the test scores.

22. Prove that if a random variable X is shifted by a constant c (i.e. let $Y = X + c$), then the variance remains the same. Provide a numerical example to illustrate your explanation.

23. Show that if a random variable is scaled by a constant a, so that $Y = aX$, then the variance is scaled by a^2; that is, $\text{Var}(Y) = a^2\,\text{Var}(X)$. Demonstrate your answer with a concrete example including calculations.

24. Derive the variance formula starting from the definition $\sigma^2 = \sum[p(x)(x-\mu)^2]$. Expand the squared term and simplify the expression to show that $\sigma^2 = E(x^2) - [E(x)]^2$.

25. Prove that if the variance of a random variable is zero, then every outcome of the random variable is identical to the mean. Provide a clear logical explanation.

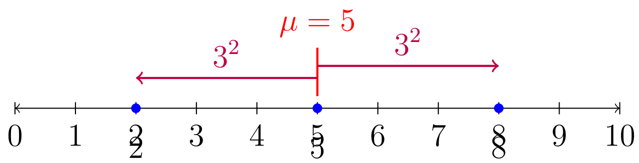
26. Consider a discrete distribution with outcomes -3, 0, 3 and 6 having probabilities 0.1, 0.4, 0.3 and 0.2 respectively. Calculate the expected value, variance and standard deviation for this distribution.

27. A distribution is partially given in the table below. Fill in the missing entries and then compute the variance and standard deviation.

x	$p(x)$	$[x-\mu]^2 \cdot p(x)$
2	0.3	?
5	?	?
8	0.4	?

Assume the probabilities sum to 1.

28. Using LaTeX and tikz, draw a number line to represent a distribution with mean 5 and outcomes at 2, 5 and 8. Clearly label the mean and show the distances (deviations) from the mean. Use squared markers (or labels) to indicate the contribution of each deviation to the variance.



29. Describe a real-world scenario where variance and standard deviation are used to analyse data. In your answer, provide a clear explanation of the context and manually produce a hand-drawn diagram (using pen and paper) that illustrates the spread of the data.

30. A random variable takes the values 5, 7, 9 and 11 with probabilities 0.1, 0.4, 0.3 and 0.2 respectively. Compute the expected value, variance and standard deviation. Then, discuss what the computed standard deviation tells you about the spread of the outcomes.