In this worksheet you will learn to define discrete random variables and understand how their distributions work. You will explore definitions, examples and properties of random variables without resorting to further analysis of expected values or variances.

Easy Questions

1. Define in your own words what a random variable is. Explain the idea that a random variable is a function mapping outcomes of a random experiment to numbers.

2. Give a simple example of a discrete random variable from everyday life. For instance, consider a coin toss where you might assign a number to head and a different number to tail. Describe your assignment.

3. Consider the experiment of rolling a standard die. If we let X be the outcome of the roll, list the possible values of X.

4. In a coin toss experiment, if we define $Y = 1$ for a head and $Y = 0$ for a tail, state whether Y is a random variable and briefly explain why.

5. Explain the difference between an individual outcome of a random experiment and the random variable that assigns numbers to outcomes.

Intermediate Questions

6. Describe the difference between a random experiment and its associated random variable. Explain how the random variable provides a numerical representation of the experiment's outcomes.

7. List three real-life examples where a discrete random variable might be used. For each example, briefly state what the sample space could be.

8. Identify two key properties that characterise discrete random variables. Explain why these properties are important when defining a random variable.

9. State what conditions a function should satisfy in order to be considered as a valid mapping from a sample space to numbers in the context of a discrete random variable.

10. Consider the experiment of tossing a coin twice. If Z is defined as the number of heads observed, write down the possible values of Z and briefly explain your reasoning.

11. For the experiment of rolling a fair die, define a random variable X as the outcome. State the sample space of the experiment and match it to the values of X.

12. Explain why the random variable itself is different from the distribution of that random variable. Use clear examples in your explanation.

13. In an experiment of drawing one card from a standard pack, define a random variable W as follows: assign Ace $= 1$, Number cards as their face value, Jack $= 11$, Queen $= 12$, and King $= 13$. List the possible numerical outputs for W.

14. Without performing any calculations, describe what it means to list the likelihood of each outcome for a discrete random variable. Explain how this list summarises the behaviour of the random variable.

15. Explain how the distribution of a discrete random variable provides a summary of the experiment. Focus on explaining how each possible value of the variable is associated with a likelihood.

16. Discuss the difference between a random variable and its realised value. Provide a simple example to illustrate your explanation.

17. Explain what is meant by the **support** of a discrete random variable. Give an example that clearly identifies the support.

18. In your own words, explain how the distribution of a discrete random variable captures the uncertainty in the outcomes of a random experiment.

19. A bag contains 3 red, 2 blue and 5 green marbles. A random variable Y is defined by assigning red $= 1$, blue $= 2$, and green $= 3$. List the possible values of Y and explain your reasoning.

20. Although not calculating specific values, explain why the sum of the probabilities assigned to the possible values of a discrete random variable must equal 1.

Hard Questions

21. Suppose X is a discrete random variable defined on the experiment of tossing two coins. Describe how you would construct a rule or a formula for X, given that you map each outcome (such as (Head, Tail)) to a number (for example, the number of heads). Provide a clear explanation of your method.

22. Consider rolling two dice and define a random variable X as the sum of the numbers shown. Without performing probability calculations, explain how the mapping from each pair of outcomes to X reflects the structure of the sample space.

23. Construct a scenario where a discrete random variable has a support that does not consist of all consecutive integers (i.e. there are gaps). Describe the random variable and explain how its support is determined.

24. For a discrete random variable X, discuss the implications of having a value in the support for which the probability is zero. What does this indicate about the random experiment and the way X is defined?

25. Compare a discrete random variable defined via a table of values to one defined by a rule or formula. Provide one example for each and discuss the advantages and disadvantages of each method.

26. Identify two common misconceptions students might have about mapping sample outcomes to numerical values in a discrete random variable. Discuss strategies that could help to clarify these misconceptions.

27. Explain how the concept of a random variable helps to simplify the analysis of a random experiment that may have many outcomes. Use an example to demonstrate your explanation.

28. Given that a random variable is a function, explain the difference between its domain and its range. Provide an example of a discrete random variable to illustrate this difference.

29. Discuss the relationship between a random variable and its realisation. How does understanding this relationship help in interpreting the outcomes of a random experiment?

30. Construct a problem scenario where a random variable is defined from a random experiment (for example, drawing lots or selecting coloured balls from a bag). Then, design a question that asks a student to list all possible values of the random variable. In your answer, explain your rationale for choosing the assignment of numbers to outcomes.