ILLAWARRA TUTORING

This worksheet focuses on calculating the expected value of a discrete random variable. You will practise finding the mean of probability distributions, deriving algebraic expressions for expected values, and solving for unknown parameters. Work through each question carefully.

Easy Questions

1. Write a short definition of the expected value (mean) of a discrete random variable.
2. A random variable X takes the values 2 and 5 with probabilities 0.3 and 0.7 respectively. Calculate $E[X]$.
3. A spinner gives outcomes 0, 1, and 2 with probabilities 0.2, 0.5, and 0.3 respectively. Find the expected value of X.
4. A random variable X has outcomes 4, 6, and 9 with probabilities 0.2, 0.5, and 0.3 respectively. Determine $E[X]$.
5. True or false: The expected value of a random variable is always equal to one of its possible outcomes. Explain your answer briefly.

Intermediate Questions

6. A random variable X has the following probability table:

Outcome	1	2	3
Probability	0.1	0.4	0.5

   Calculate $E[X]$.

7. A game offers winnings X dollars with outcomes -5, 0, and 10 having probabilities 0.2, 0.5, and 0.3 respectively. Compute the expected winnings.
8. In a lottery, the possible prize amounts are 0, 20, and 100 dollars with probabilities 0.8, 0.15, and 0.05 respectively. Find the expected prize value.
9. A random variable X takes the values 3, 7, and 12 with probabilities 0.3, 0.4, and 0.3 respectively. Calculate $E[X]$.
10. Create a table listing the outcomes 1, 3, 5, and 7 with corresponding probabilities 0.1, 0.3, 0.4, and 0.2. Use your table to compute $E[X]$.

11. A random variable X takes values -2, 0, and 2 with probabilities 0.3, 0.4, and 0.3 respectively. Determine its expected value.

12. A machine dispenses treat coins such that a random variable X takes values 0, 1, 2, and 3 with probabilities 0.1, 0.5, 0.3, and 0.1 respectively. Calculate $E[X]$.

13. In a game, you can score 10, 20, or 30 points with probabilities 0.2, 0.5, and 0.3 respectively. Find the expected score.

14. A random variable Y has outcomes 5, 8, and 9 with probabilities 0.3, 0.3, and 0.4 respectively. Calculate $E[Y]$.

15. A bag contains 3 red balls and 7 blue balls. If you randomly select one ball, let the random variable X equal 1 if it is red and 0 if blue. Determine $E[X]$.

16. In a raffle, a ticket wins 10 dollars with probability 0.05, 5 dollars with probability 0.1, or nothing with probability 0.85. Calculate the expected winning.

17. A fair coin is tossed once. Let the random variable X be 1 if heads occurs and 0 if tails. What is $E[X]$?

18. In a game using a single die, if an even number appears you win that number of points; if an odd number appears you lose that number of points. Assuming each outcome has probability $\frac{1}{6}$, calculate the expected value of your points.

19. A scratch card game gives winnings of 2, 5, or 20 dollars with probabilities 0.6, 0.3, and 0.1 respectively. Determine the expected win amount.

20. A vending machine awards points based on the snack you get. The points are 1, 2, and 3 with probabilities 0.3, 0.4, and 0.3 respectively. Compute $E[X]$.

Hard Questions

21. Given a random variable X with outcomes 2, 4, and 6 with probabilities 0.3, 0.4, and 0.3 respectively, let $Y = X+3$. Calculate $E[Y]$ and verify that $E[Y] = E[X]+3$.

22. Let a random variable X take values a, b, and c with probabilities p, q, and $1-p-q$ respectively. Express the expected value $E[X]$ in terms of a, b, c, p and q.

23. A random variable X takes values 1, 2, and 3 with probabilities p, $2p$, and $1-3p$ respectively. First express $E[X]$ in terms of p, and then determine the value of p if $E[X] = 2.5$.

24. Suppose a random variable Y takes values 0, 3, and 5 with probabilities q, 0.4, and $0.6-q$ respectively. Write an expression for $E[Y]$ in terms of q and simplify as much as possible.

25. A deck contains cards numbered 1 to 5 where the probability of drawing a card is proportional to its number. Determine the probability distribution and calculate the expected card number.

26. A factory produces items with quality scores 1, 2, 3, and 4. The probability of an item scoring a number is proportional to that number. Determine the probability distribution and compute the expected quality score.

27. A random variable X takes the values 2, 4, and 6 with weights 2^1, 2^2, and 2^3 respectively. First, find the normalising constant for the probabilities and then calculate $E[X]$.

28. A fair coin is tossed three times. Without referencing any formula, list all possible outcomes for the number of heads and calculate the expected number of heads.

29. In a betting game you pay \$2 to play. A wheel is spun which returns a multiplier of 0, 1, 2, or 3 with equal probability. Your winning is calculated as 2 times the multiplier. Determine the expected net profit from playing the game.

30. A random variable X takes the values 1, 2, and 3 with probabilities 0.2, r, and $0.8 - r$ respectively. Given that $E[X] = 2.4$, find the value of r.