ILLAWARRA TUTORING

This worksheet focuses on calculating the mean or expected value for a discrete random variable. You will practise computing the weighted average outcome where each value is multiplied by its probability. Work through each question carefully and show your workings.

Easy Questions

1. Calculate the expected value of the random variable X that takes the values $1, 2, 3$ with probabilities $\frac{1}{4}, \frac{1}{2}$ and $\frac{1}{4}$ respectively.
2. A random variable Y takes values 0 and 5 with probabilities 0.3 and 0.7 respectively. Determine $E(Y)$.
3. Given the probability distribution: $X : 2\,(0.2),\ 4\,(0.5),\ 6\,(0.3)$. Calculate the expected value of X.
4. Write the formula for the expected value of a discrete random variable X and briefly explain what it represents.
5. The random variable Z has outcomes $-1, 0, 1$ with probabilities $0.2, 0.5, 0.3$ respectively. Confirm that the probabilities sum to 1 and compute $E(Z)$.

Intermediate Questions

6. Compute the expected value of a fair die roll where the outcomes are $1, 2, 3, 4, 5, 6$.
7. In a coin toss game, you earn 3 dollars for heads and 1 dollar for tails. If the coin is fair, what is the expected earning per toss?
8. A random variable V is defined by the following table:

x	0	2	5
$p(x)$	0.4	0.35	0.25

   Compute $E(V)$.
9. Explain why higher probability outcomes influence the expected value more than lower probability outcomes. Illustrate your explanation with an example.
10. A spinner is divided into 4 equal sectors with values $1, 2, 3, 4$. Draw a diagram show the spinner and then calculate the expected value of the outcome.

11. In a raffle, a ticket wins 10 dollars with probability 0.1 and nothing with probability 0.9. What is the expected winning if you buy a ticket?

12. A game costs 2 dollars to play and you have a chance to earn 8 dollars with probability 0.25 and 0 dollars otherwise. Calculate the expected net gain (or loss) per game.

13. Consider a process where you perform an experiment with 3 outcomes giving points $0, 1$, and 2 with probabilities $0.2, 0.5$, and 0.3 respectively. Find the expected points per experiment.

14. Explain in your own words what is meant by the expected value of a discrete random variable and why it is considered a long-run average.

15. A random variable W has outcomes $2, 4, 7$ with probabilities $\frac{1}{3}, \frac{1}{6}, \frac{1}{2}$ respectively. First, verify that the probabilities total 1, then determine $E(W)$.

16. The random variable U can take values $1, 3, 5$ with probabilities $p, 0.5$ and 0.2 respectively. If $E(U) = 3.4$, determine the value of p.

17. In an insurance scheme, a claim pays -100 dollars (a loss) with probability 0.05 and 0 dollars with probability 0.95. Calculate the expected value of a claim.

18. Given a random variable X with the following distribution:

x	0	5	10
$p(x)$	0.4	q	0.3

If q is unknown and $E(X) = 6$, determine the value of q.

19. In a simple lottery, a ticket has a 0.02 chance of winning 50 dollars, and a 0.98 chance of winning nothing. Compute the expected winning of one ticket.

20. Two discrete random variables A and B are defined as follows:

x	0	10
$p_A(x)$	0.7	0.3

and

x	2	8
$p_B(x)$	0.4	0.6.

Calculate $E(A)$ and $E(B)$ and state which game would be preferable if you wished to maximise your returns.

Hard Questions

21. Let X and Y be two discrete random variables with finite outcomes. Prove that $E(X + Y) = E(X) + E(Y)$ by writing out the sum in full.

22. Given a discrete random variable X with values x_i and probabilities p_i, show that the expected value of $g(X) = 2x + 3$ is $2E(X) + 3$.

23. A random variable Z takes the values $-3, 0, 4$ with probabilities $0.3, 0.4$, and 0.3 respectively. Calculate $E(Z)$ and explain any significance of a negative expected value in context.

24. In a game, you pay 5 dollars to play. The game pays out according to the following distribution: 15 dollars with probability 0.1, 7 dollars with probability 0.2, and 0 dollars with probability 0.7. Calculate the expected payout and decide whether the game is fair.

25. Consider a two-stage experiment. In the first stage, a coin is flipped (heads gives a value of 4 and tails 0). If heads, a six-sided die is rolled and its outcome is added to 4; if tails, no die is rolled. Find the expected total score of this experiment.

26. A random variable X takes values $1, 2, 3$ with probabilities $p, 2p$, and $1-3p$ respectively. If $E(X) = 2.2$, find the value of p.

27. A game offers payoffs of $2, 5$, and 12 dollars with probabilities $0.5, 0.3$, and q respectively. If the game is known to be fair (i.e. zero net gain when the cost of 7 dollars is included), determine q.

28. A shop sells lottery scratch cards. It is known that a scratch card yields a prize of 20 dollars with probability 0.04, 5 dollars with probability 0.15 and no prize with probability 0.81. If each scratch card costs 3 dollars, compute the expected profit (or loss) per scratch card for a customer.

29. Two games are offered: In Game I, you earn x dollars if a coin toss is heads and 0 dollars if tails. In Game II, you earn $0.5x$ dollars if heads and $2x$ dollars if tails. For which value(s) of x are the expected values of the two games equal? Assume a fair coin in both games.

30. A carnival game works as follows: You pay 4 dollars to play. You then spin a wheel that can land on any of the numbers $1, 2, 3, 4, 5$ with probabilities $0.1, 0.2, 0.3, 0.25$, and 0.15 respectively. You win an amount in dollars equal to twice the number spun. Calculate the expected winning and hence, determine the expected net gain or loss.