This worksheet focuses on calculating the expected value of a discrete random variable so you know what outcome to anticipate on average.

Easy Questions

1. Calculate the expected value of a random variable X that takes the values 1, 2, 3 with probabilities 0.2, 0.3, 0.5 respectively.
2. A fair coin is tossed and a random variable Y is defined as follows: $Y = 0$ for tails and $Y = 1$ for heads. Calculate the expected value of Y.
3. A random variable Z takes values -1, 0, 1 with probabilities 0.3, 0.4, 0.3 respectively. Find the expected value of Z.
4. Write the formula for the expected value $E[X]$ of a discrete random variable X in summation notation.
5. A fair six-sided die has outcomes 1, 2, 3, 4, 5, 6 each with equal probability. Calculate the expected value of the outcome.

Intermediate Questions

6. A random variable X takes values 2, 4, 6, 8 with probabilities 0.1, 0.2, 0.3, 0.4 respectively. Compute $E[X]$.
7. For a weighted coin toss, let a random variable W be defined by $W = 0$ with probability 0.6 and $W = 10$ with probability 0.4. Calculate the expected value $E[W]$.
8. A random variable V takes the values -3, 1, 5 with probabilities 0.2, 0.5, 0.3 respectively. Determine $E[V]$.
9. A lottery ticket yields gains given by G where $G = -5$, 0, 15 with corresponding probabilities 0.25, 0.5, 0.25. Calculate the expected gain $E[G]$.
10. In a raffle, the prize amounts are 50, 20, 0 with probabilities 0.1, 0.2, 0.7 respectively. Compute the expected prize amount.
11. Consider a random variable X that takes the values 3, 7, 9 with probabilities 0.3, 0.4, 0.3 respectively. Find $E[X]$.
12. A random variable Y takes values 10, 20, 30, 40 with probabilities 0.1, 0.2, 0.3, 0.4 respectively. Calculate the expected value $E[Y]$.

13. In a game, the score S can be $0,\ 100,\ 200$ with probabilities $0.5,\ 0.3,\ 0.2$ respectively. Determine $E[S]$.

14. A random variable N representing the number of successes in a trial takes values $0,\ 1,\ 2$ with probabilities $0.1,\ 0.7,\ 0.2$ respectively. Compute the expected value $E[N]$.

15. A loaded six-sided die has outcomes $1,\ 2,\ 3,\ 4,\ 5,\ 6$ with probabilities $0.1,\ 0.15,\ 0.2,\ 0.25,\ 0.2,\ 0.1$ respectively. Find $E[X]$ where X is the outcome.

16. A random variable A takes values $2,\ 4,\ 8,\ 16$ with probabilities $0.4,\ 0.3,\ 0.2,\ 0.1$ respectively. Calculate $E[A]$.

17. A profit random variable P has outcomes $-10,\ 0,\ 10$ with probabilities $0.3,\ 0.4,\ 0.3$ respectively. Find the expected profit $E[P]$.

18. In a contest, the score Q can be $5,\ 10,\ 15,\ 20$ with probabilities $0.2,\ 0.3,\ 0.3,\ 0.2$ respectively. Calculate $E[Q]$.

19. A random variable X takes values $-2,\ 3,\ 7,\ 10$ with respective probabilities $0.1,\ 0.4,\ 0.3,\ 0.2$. Compute the expected value $E[X]$.

20. A spinner yields the outcomes $0,\ 5,\ 10,\ 15$ each with equal probability. Determine the expected value of the winnings.

Hard Questions

21. A random variable X takes values $1,\ 2,\ 3$ with probabilities proportional to $1,\ 2,\ 4$. That is, $P(X = 1) = c$, $P(X = 2) = 2c$ and $P(X = 3) = 4c$. Determine the value of c and calculate $E[X]$.

22. A random variable X has a probability mass function given by $P(X = 1) = a$, $P(X = 2) = 2a$, and $P(X = 3) = 1 - 3a$ where $0 < a < \frac{1}{3}$. If $a = 0.1$, compute the expected value $E[X]$.

23. Let Y be a discrete random variable that takes values $-2,\ 0,\ 2$ with probabilities $0.2,\ 0.5,\ 0.3$ respectively. Define $X = 2Y + 1$. Calculate the expected value $E[X]$ using the linearity of expectation.

24. A random variable X takes values $1,\ 2,\ 3,\ 4,\ 5$ with probabilities given by $P(X = x) = \frac{x}{15}$. Compute $E[X]$.

25. A random variable X takes values $0,\ 1,\ 3,\ 7$ with probabilities proportional to the value itself (with the convention that $P(X = 0) = 0$). First, determine the normalising constant and then calculate $E[X]$.

26. Let X have outcomes $1,\ 2,\ 3$ with probabilities given by $P(X = x) = \dfrac{x^2}{\sum_{x=1}^{3} x^2}$.

    Compute the expected value $E[X]$.

27. A spinner shows the numbers 1, 2, 3, 4 with equal probability. A player's winning amount is given by $S = X^2$ where X is the number spun. Find the expected winning amount $E[S]$.

28. A random variable X takes values $-3, -1, 2, 4$ with probabilities 0.2, p, p, 0.3 respectively. First, determine the value of p and then compute $E[X]$.

29. Let X be a random variable that takes values n where $n = 1, 2, 3, 4, 5$ with probabilities $P(X = n) = k(6 - n)$. Determine the constant k and calculate $E[X]$.

30. Prove that if a random variable X takes a constant value c with probability 1, then the expected value $E[X]$ is equal to c.