ILLAWARRA TUTORING

In this worksheet you will practise constructing and interpreting probability distribution tables and graphs for various scenarios. Work through each question carefully and check that your distributions obey all the axioms of probability.

Easy Questions

1. Construct a probability distribution table for a fair coin toss. Let the random variable X denote the number of heads obtained.

2. A spinner is divided into three equal regions labelled 1, 2 and 3. Construct a probability distribution table for the outcome of one spin.

3. For a fair six-sided die roll, let X be the number showing. Construct the probability distribution table.

4. A bag contains 4 blue marbles and 6 red marbles. One marble is drawn at random. Let the random variable X equal 1 if a blue marble is drawn and 0 if a red marble is drawn. Construct the probability distribution table.

5. Two fair coins are tossed. Let the random variable X equal the number of heads obtained. Construct the probability distribution table and then, using pen and paper, draw a bar graph representing the distribution. Be sure to label your axes.

Intermediate Questions

6. For two independent fair coin tosses, let X be the number of heads obtained. Construct the probability distribution table.

7. A deck consists of five cards labelled 1, 2, 3, 4 and 5, each equally likely to be drawn. Construct the probability distribution table for the number on the drawn card.

8. A bag contains 3 green balls and 2 yellow balls. Two balls are drawn without replacement. Let X be the number of green balls drawn. Construct the probability distribution table for X.

9. In a board game, a wheel divided into 4 equal sections numbered 1 to 4 is spun once. Construct the probability distribution table for the number spun.

10. A fair coin is tossed three times. Let X be the number of tails observed. Construct the probability distribution table for X.

11. For a fair die roll, define the random variable Y as 1 if the outcome is even and 0 if the outcome is odd. Construct the probability distribution table for Y.

12. A jar contains 10 tickets: 7 numbered 1 and 3 numbered 2. One ticket is selected at random. Construct the probability distribution table for the number on the ticket.

13. A game machine dispenses a prize according to the number on a drawn ball. The outcomes and their probabilities are: 1 with probability 0.1, 2 with probability 0.2, 3 with probability 0.3, 4 with probability 0.25, and 5 with probability 0.15. Construct the probability distribution table.

14. In a multiple-choice question with 4 possible answers (only one correct), a student guesses randomly. Let the random variable X equal 1 for a correct guess and 0 for an incorrect guess. Construct the probability distribution table.

15. A toy machine randomly generates a digit from 0 to 3. The probabilities are: for 0, 0.4; for 1, 0.3; for 2, 0.2; for 3, 0.1. Construct the probability distribution table for the digit.

16. A box contains 3 cards numbered 1, 2 and 3 with non-equal probabilities: 1 occurs with probability 0.2, 2 with probability 0.5, and 3 with probability 0.3. Construct the probability distribution table for the drawn card.

17. A bag contains 5 coins, of which 2 are double-headed and 3 are fair coins. A coin is selected at random and flipped once. Let X be the number of heads observed. Construct the probability distribution table for X.

18. Two fair dice are rolled. Let X be the sum of the numbers shown. Construct the probability distribution table for X. (List only the outcomes that have positive probability.)

19. An urn contains balls numbered 2, 4, 6 and 8 with corresponding probabilities 0.1, 0.2, 0.3 and 0.4. Construct the probability distribution table for the number drawn.

20. A student tosses a fair coin and rolls a fair die simultaneously. Define the random variable X as follows: if the coin lands tail, then X equals the die outcome; if the coin lands head, then X equals the die outcome plus 1. Construct the probability distribution table for X.

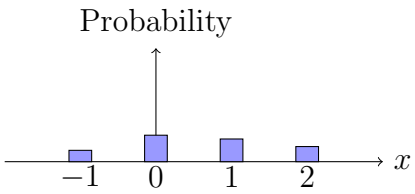
Hard Questions

21. Given the probability distribution table below for X:

x	$P(X = x)$
0	0.1
1	0.2
2	0.4
3	0.3

Construct a bar graph by hand that represents this distribution. Label the horizontal axis with x and the vertical axis with probability.

22. The diagram below represents the probabilities of a discrete random variable X with outcomes -1, 0, 1 and 2.



Using the diagram, construct a possible probability distribution table for X.

23. A probability distribution table for X is given below:

x	$P(X = x)$
1	0.05
2	0.15
3	0.5
4	0.3

Determine whether this table defines a valid probability distribution. In your answer, explain the conditions that every probability distribution must satisfy.

24. Construct a probability distribution table for a random variable Y that has 5 outcomes with probabilities in the ratio $1 : 2 : 3 : 2 : 1$. Then, using pen and paper, draw a bar graph to represent your distribution. (Hint: First find the normalising constant.)

25. A bag contains balls numbered 1, 2, 3 and 4. The probability of drawing a ball is proportional to the number on the ball. Construct the probability distribution table for the number drawn and then, using pen and paper, draw the corresponding bar graph.

26. Consider a random variable X with outcomes 10, 20 and 30. Suppose the probabilities are given by a linear function $f(x) = a + bx$ such that

$$f(10) = 0.2 \quad \text{and} \quad f(30) = 0.6.$$

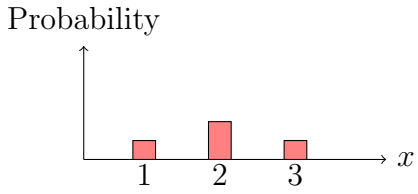
Compute $f(20)$. Notice that the sum of $f(10)$, $f(20)$ and $f(30)$ is not 1. Normalise these probabilities to form a valid probability distribution table.

27. A probability distribution table for the number of customers arriving at a store in one hour is partially given as follows:

Customers	$P(X = x)$
0	0.1
1	0.3
2	p
3	0.2

Find the value of p that makes this a valid probability distribution. Then, sketch a bar graph (using pen and paper) to represent the complete distribution.

28. Examine the probability distribution graph provided in the diagram below.



From the graph, write out a possible probability distribution table for X. In addition, state two essential properties that every probability distribution must satisfy.

29. Create a real-world scenario (other than those mentioned earlier) where the outcomes have a non-uniform probability distribution. Construct the probability distribution table for your scenario and draw the corresponding bar graph (using pen and paper). Explain briefly why your scenario leads to a non-uniform distribution.

30. A teacher gives a quiz where each student can receive a grade of A, B or C. The probabilities of receiving the grades are in the ratio $1 : 3 : 2$. Construct the probability distribution table for the grades and then draw a bar graph representing your distribution. Finally, verify that the probabilities sum to 1.