ILLAWARRA TUTORING

This worksheet focuses on the binomial distribution. In this unit you will explore situations that consist of a fixed number of independent trials, each with the same probability of success, and learn how to calculate probabilities using the formula $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$.

Easy Questions

1. State three conditions that must be met for a probability experiment to be classified as a binomial experiment.
2. Write down the formula for the probability of exactly k successes in n independent trials, each with probability p of success.
3. A fair coin is tossed four times. Calculate the probability of getting exactly 2 heads. (Assume heads is the success.)
4. Explain why the combination term $\binom{n}{k}$ is used when computing the probability in a binomial distribution.
5. In your own words, explain what is meant by the trials being independent in a binomial experiment.

Intermediate Questions

11. In an experiment with $n = 10$ trials and probability of success $p = 0.3$, calculate the probability of obtaining exactly 4 successes.
12. For a binomial distribution with $n = 20$ and $p = 0.4$, calculate the expected value (mean).
13. For a binomial experiment with $n = 15$ trials and probability $p = 0.5$, calculate the variance.
14. In a binomial experiment with $n = 8$ and $p = 0.25$, determine the probability of obtaining at most 2 successes.
15. In an experiment with $n = 12$ trials and $p = 0.6$, calculate the probability of obtaining at least 9 successes.

16. Compute $\binom{7}{3}$ and explain its significance with respect to counting the number of ways to achieve successes.

17. A local sports team has a probability of 0.6 to win any match. If they play 5 matches, compute the probability that they win exactly 3 matches.

18. In a class, each student has a probability of 0.8 to pass an exam. Using a binomial model with $n = 10$ students, find the probability that exactly 7 students pass.

19. A factory produces components such that each component has a 0.95 probability of functioning. If 20 components are selected, compute the probability that at most 2 components fail.

20. In a scenario with $n = 6$ trials and probability of success $p = 0.4$, determine the probability of obtaining no more than 2 successes.

21. A student takes a quiz made up of 5 true-false questions. If the probability of guessing correctly on each question is p, and the probability of getting exactly 3 questions correct is 0.3125, show how one might set up an equation to solve for p.

22. Explain why the binomial distribution is symmetric when the probability of success is 0.5. Provide a brief justification.

23. Discuss how increasing the number of trials n in a binomial experiment (with fixed p) affects the shape of the probability distribution.

24. Using a calculator or cumulative binomial probability table, calculate the probability of obtaining at least 4 successes in 8 trials with $p = 0.3$.

25. In an experiment with $n = 10$ and $p = 0.2$, compute the probability of obtaining either 0 or 10 successes. Explain why these probabilities might be very different.

Hard Questions

21. Derive the formulas for the mean and variance of a binomial distribution by considering it as the sum of n independent Bernoulli trials.

22. Prove mathematically that a binomial distribution with $p = 0.5$ is symmetric about its mean.

23. Using the complement rule, show that the probability of at least one success in 12 trials with $p = 0.1$ is $1 - (0.9)^{12}$. Explain each step.

24. A quality control inspector tests 20 items with a defect probability of 0.05 per item. Calculate the probability that at least one item is defective, and outline your reasoning.

25. A multiple-choice test consists of 10 questions, each with 4 options and one correct answer. Calculate the probability of guessing correctly on at least 6 questions.

26. Prove that the sum of the probabilities for all possible outcomes in a binomial distribution (from $k = 0$ to $k = n$) equals 1.

27. Consider a binomial random variable with $n = 12$. If its variance is given as 3, find the corresponding probability of success p. Show your working.

28. A basketball player has a free-throw success rate of 0.75. If she takes 12 free throws, calculate the probability that she makes more than 10 free throws. Outline your steps.

29. For fixed n and k, show how you can demonstrate that the probability $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$ is maximised when $p = \frac{k}{n}$. (A complete formal proof is not required; an outline of the idea is sufficient.)

30. Suppose a binomial random variable X with $n = 8$ satisfies $P(X = 3) = 0.21875$. Set up and solve the equation to determine the value of p.