In this worksheet you will learn to apply discrete probability distributions, particularly the binomial distribution, to solve practical problems in various fields. Read each question carefully and show all your working.

Easy Questions

1. Given $n = 3$ independent trials with a success probability $p = 0.5$ on each trial, calculate $P(X = 2)$.

2. A factory has a probability $p = 0.1$ that an item is defective. If 5 items are produced, determine $P(\text{exactly 1 defective})$.

3. For a binomial experiment with $n = 4$ and $p = 0.25$, find the mean (expected value) $E(X)$.

4. For the same experiment described in Question 3, calculate the variance $\text{Var}(X)$.

5. Read the following scenario and state whether it is appropriate to model it with a binomial distribution. Explain briefly: A teacher randomly selects 6 students from a class of 30 to determine if they have completed their homework.

Intermediate Questions

6. A factory produces items with a defect probability of 0.05 per item. If 20 items are produced, compute $P(X = 1)$.

7. In a binomial experiment with $n = 15$ and $p = 0.3$, determine $P(X \leq 2)$.

8. A fair coin is tossed 7 times. Calculate the probability that exactly 4 heads occur.

9. For independent trials with $n = 5$ and success probability $p = 0.2$, find $P(X \geq 1)$.

10. In an exam, the chance of correctly answering a question is 0.6. If a student answers 10 questions, determine $P(X = 7)$ using the binomial distribution.

11. For a binomial distribution with $n = 12$ and $p = 0.25$, compute the expected number of successes.

12. Using the parameters from Question 11, calculate the variance $\text{Var}(X)$.

13. In a sales scenario, if the probability of making a sale is 0.15 and a salesperson makes 30 calls, compute $P(X = 5)$.

14. In a quality test where an item is faulty with probability 0.1, if 12 items are tested, determine $P(X = 0)$.

15. A tennis player wins a point with probability 0.55. In a game of 8 points, find $P(X \geq 6)$.

16. In a multiple-choice exam with 4 options per question, a student guesses randomly. If there are 10 questions, compute $P(X = 3)$, where $p = 0.25$.

17. An archer has a probability of 0.7 to hit the target on any shot. In 5 shots, determine the probability that the archer misses every time.

18. For a binomial experiment with $n = 10$ and an unknown probability p, if the expected value is 6, find p.

19. In a shooting contest, the probability of hitting the mark is 0.8. Over 15 attempts, compute $P(X < 12)$.

20. In a game where the probability to score on a turn is 0.4, and the player takes 10 turns, calculate $P(3 \leq X \leq 5)$.

Hard Questions

21. A network experiences failures independently with a probability of 0.02 per day. Over 50 days, calculate $P(X = 2)$.

22. In a card game, the probability of winning a round is 0.35. If 20 rounds are played, determine $P(X > 10)$.

23. A newborn screening test correctly detects a disease with probability 0.98 when the disease is present. If 12 affected newborns are tested, compute $P(X \geq 11)$.

24. In a lottery game, the chance of winning a small prize on a ticket is 0.05. If 40 tickets are purchased, determine $P(X = 3)$.

25. An assembly line produces items that are non-defective with a probability of 0.97. For a batch of 100 items, calculate $P(X \leq 95)$ where X is the number of non-defective items.

26. A basketball player has a free-throw success rate of 0.8. If he attempts 15 free throws, compute $P(X = 14)$.

27. For a binomial distribution with $n = 12$ and success probability p, find the value of p if $P(X = 5) = \frac{1}{2}P(X = 4)$.

28. In a research study, the chance that a trial has a positive outcome is 0.3. With 25 independent trials, determine $P(X = 8)$.

29. For a binomial distribution with $n = 18$ and $p = 0.4$, calculate $P(X \leq 8)$.

30. A quality control inspector finds that each item has a probability of 0.06 of being faulty. For a sample of 50 items, compute $P(2 \leq X \leq 5)$, where X is the number of faulty items.