ILLAWARRA TUTORING

This worksheet is designed to help you learn to apply discrete probability distributions to solve practical problems in various fields. Work through the questions carefully and ensure you show all steps in your calculations.

Easy Questions

1. A company produces light bulbs. The number of defective bulbs in a package is given by the following probability distribution:
   $P(X = 0) = 0.8, \quad P(X = 1) = 0.15, \quad P(X = 2) = 0.05.$
   Find the probability that a package contains no defective bulbs.

2. In a small garage, the number of cars arriving in an hour follows the distribution:
   $P(\text{Cars} = 0) = 0.1, \quad P(\text{Cars} = 1) = 0.4, \quad P(\text{Cars} = 2) = 0.3, \quad P(\text{Cars} = 3) = 0.2.$
   Determine the probability that more than 1 car arrives in a given hour.

3. A multiple-choice question has 4 answer options and only 1 correct answer. If a student guesses randomly on 4 independent questions, what is the probability of getting exactly 2 correct answers? (Assume each question is independent.)

4. A fair six-faced die is rolled once. Every face has an equal probability. Calculate the expected value of the outcome.

5. Consider a discrete random variable given by:
   $P(Y = 1) = 0.5, \quad P(Y = 2) = 0.3, \quad P(Y = 3) = 0.2.$
   Compute the variance of Y.

Intermediate Questions

6. A light bulb manufacturer has a defect rate of 0.1. If 10 bulbs are selected randomly, calculate the probability that exactly 1 bulb is defective.

7. In a call centre, a particular agent answers a call with probability 0.2. If 8 calls are made, compute the probability that the agent receives at least 2 calls.

8. In a lottery, each ticket has a 0.05 probability of winning a prize. If you buy 20 tickets, what is the probability that you win at least one prize?

9. A student responds to a 5-question multiple-choice exam by guessing. Each question has 4 choices so that the probability of a correct answer is 0.25. Determine the probability that the student gets exactly 3 answers correct.

10. A survey finds that 30% of people prefer tea. If 15 people are surveyed, calculate the probability that exactly 5 people prefer tea.

11. The number of system failures in a day is given by the following distribution:
$P(X = 0) = 0.5, \quad P(X = 1) = 0.3, \quad P(X = 2) = 0.15, \quad P(X = 3) = 0.05.$
Determine the probability that there will be at least 2 failures in a day.

12. In a quality test, the probability that a randomly selected item is defective is 0.02. For a batch of 100 items, calculate the probability that at most 3 items are defective.

13. An archer has a 0.7 probability of hitting the target with each shot. In 12 shots, calculate the probability that he hits the target more than 8 times.

14. In a dice game, the probability of rolling a 6 (event A) is $\frac{1}{6}$. If the die is rolled 18 times, determine the probability that event A occurs exactly 3 times.

15. In a city, the probability that a car is electric is 0.15. If 20 cars are parked in a lot, compute the probability that exactly 4 cars are electric.

16. A survey indicates that 40% of households have pets. If 10 households are randomly selected, calculate the probability that fewer than 3 households have pets.

17. In a new drug trial, the probability a patient responds favourably is 0.6. If 8 patients are treated, determine the probability that exactly 5 patients respond favourably.

18. A student guesses answers on a 10-question true-false exam. Compute the probability that the student gets at least 7 questions correct.

19. A company sends out promotional emails with a response probability of 0.25 per email. If 12 emails are sent, calculate the probability that exactly 4 responses are received.

20. A sports team has a 0.55 probability of winning any match. In a tournament of 6 matches, compute the probability that the team wins exactly 4 matches.

Hard Questions

21. A factory produces items with a defective rate of 0.01. If a sample of 200 items is inspected, calculate the probability that at least 5 items are defective.

22. In a call centre, each call is answered successfully with a probability of 0.3. For 15 independent calls, compute the probability that the number of successful responses is between 4 and 7 (inclusive).

23. An archer shoots 20 arrows with a probability of 0.45 of hitting the target on each shot. Calculate the probability that the archer scores exactly 10 hits.

24. At a retail store, each customer makes a purchase with a probability of 0.15. If 50 customers enter the store, determine the probability that more than 10 customers make a purchase.

25. A student takes a test consisting of 25 multiple-choice questions by guessing. Each question has 5 options, so the probability of answering correctly is 0.2. Calculate the probability that the student gets exactly 8 questions correct.

26. A chain store's weekly customer visit count is modelled by the distribution:
$P(\text{visits} = 50) = 0.3, \quad P(\text{visits} = 60) = 0.5, \quad P(\text{visits} = 70) = 0.2.$
Calculate the expected number of weekly visits.

27. Using the same distribution in Q26, compute the variance of the weekly customer visits.

28. A game show contestant is given 5 independent challenges, with a probability of 0.4 of winning each challenge. Determine the probability that the contestant wins at least 3 challenges.

29. A computer server experiences crashes in a week according to the following distribution:
$P(0 \text{ crashes}) = 0.6, \quad P(1 \text{ crash}) = 0.3, \quad P(2 \text{ crashes}) = 0.08, \quad P(3 \text{ crashes}) = 0.02.$
Assuming that the crash occurrences are independent across weeks, calculate the probability that there will be 0 crashes in two consecutive weeks.

30. A manufacturing process produces goods with the number of defects per unit modelled by the distribution:
$P(\text{defects} = 0) = 0.7, \quad P(\text{defects} = 1) = 0.2, \quad P(\text{defects} = 2) = 0.1.$
If 5 independent units are produced, calculate the probability that the total number of defects is exactly 1. (Hint: Consider which unit could contribute the single defect while the others have none.)