ILLAWARRA TUTORING

In this worksheet you will explore fractional indices and learn to convert between radical and exponential forms. You will practice converting expressions, simplifying using index laws and solving equations involving fractional exponents.

Easy Questions

1. Convert the radical expression $\sqrt{a}$ to an expression with a fractional index.
2. Write the expression $a^{\frac{3}{2}}$ in radical form.
3. Evaluate $\left(16^{\frac{1}{4}}\right)^2$.
4. Express the radical $\sqrt[3]{b^2}$ as an expression with a fractional index.
5. Simplify $\left(x^5\right)^{\frac{1}{5}}$.

Intermediate Questions

6. Evaluate $27^{\frac{2}{3}}$.
7. Simplify $8^{\frac{2}{3}} \times 8^{\frac{1}{3}}$.
8. Simplify $\dfrac{y^{\frac{7}{3}}}{y^{\frac{4}{3}}}$.
9. Write $x^{\frac{3}{4}}$ in radical form.
10. Evaluate $16^{\frac{3}{4}}$.
11. Solve for x in the equation: $x^{\frac{2}{3}} = 64$.
12. Express $32^{\frac{2}{5}}$ in radical form and simplify.
13. Write $a^{\frac{5}{6}}$ in radical form.
14. Compute $\left(81^{\frac{1}{4}}\right)^2$.
15. Simplify $16^{\frac{3}{4}} \times 16^{\frac{1}{4}}$.
16. Express $\sqrt[3]{x^2} \times \sqrt[3]{x^4}$ as a single term with a fractional exponent.
17. Write $\sqrt[5]{y^3}$ in exponential form.

18. Simplify $a^{\frac{1}{2}} \times a^{\frac{1}{3}}$.

19. Solve for x if $4^{\frac{x}{2}} = 8$.

20. Simplify $\dfrac{9^{\frac{1}{2}}}{27^{\frac{1}{3}}}$.

Hard Questions

21. Prove that for any positive real number a and integers m, n with $n > 0$,

$$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m.$$

Provide a clear explanation.

22. Solve for x in the equation

$$\left(\frac{3x}{2}\right)^{\frac{4}{3}} = 81.$$

23. Prove that

$$a^{\frac{3}{4}} \times a^{\frac{1}{4}} = a,$$

and explain each step in your reasoning.

24. If

$$\left(x^{\frac{5}{2}}\right)^{-\frac{2}{5}} = x^c,$$

find the value of c.

25. Solve for x in

$$\left(x^{\frac{2}{3}}\right)^3 = 64.$$

26. Simplify

$$\left(16^{\frac{1}{4}} \times 4^{\frac{1}{2}}\right)^2.$$

27. Simplify

$$\left(\frac{a^{\frac{1}{3}}}{a^{\frac{2}{3}}}\right)^3.$$

28. Express

$$\left(\sqrt[6]{b^4}\right)^3$$

in simplest exponential form.

29. If

$$27^{\frac{m}{3}} = 9,$$

determine the value of m.

30. Write the expression

$$\sqrt[4]{16x^8}$$

in exponential form and simplify it.