In this worksheet you will explore fractional indices and learn to convert between radical and exponential forms. You will practise rewriting roots as rational exponents, performing operations with these exponents, and solving equations that involve fractional indices.

Easy Questions

1. Write $\sqrt{x}$ in exponential form.

2. Write $\sqrt[3]{y}$ in exponential form.

3. Evaluate $27^{\frac{1}{3}}$.

4. Write $a^{\frac{3}{4}}$ in radical form.

5. If $b^{\frac{1}{2}} = 4$, find the value of b.

Intermediate Questions

6. Simplify $x^{\frac{1}{2}} \times x^{\frac{1}{3}}$.

7. Evaluate $16^{\frac{3}{4}}$.

8. Write $\sqrt[3]{a^2}$ in fractional exponent form.

9. Simplify $\dfrac{x^{\frac{3}{4}}}{x^{\frac{1}{4}}}$.

10. If $a^{\frac{1}{3}} = 5$, find the value of a.

11. Write $\left(\dfrac{a}{b}\right)^{\frac{1}{2}}$ in radical form.

12. Solve for x in $(2x)^{\frac{3}{2}} = 16$ (assume $x > 0$).

13. Simplify $9^{\frac{1}{2}} \times 9^{\frac{1}{4}}$.

14. Write $m^{\frac{7}{5}}$ in radical form.

15. Simplify $\left(a^{\frac{1}{3}}\right)^6$.

16. Evaluate $\left(27^{\frac{1}{3}}\right)^2$.

17. Simplify $\left(x^{\frac{2}{3}}y^{\frac{1}{3}}\right)^3$.

18. Evaluate $4^{\frac{1}{2}} \times 8^{\frac{1}{3}}$.

19. Write $x^{-\frac{1}{2}}$ in radical form.

20. If $k^{\frac{2}{3}} = 16$, find k (assume $k > 0$).

Hard Questions

21. Solve for x in $x^{\frac{3}{2}} = 64$.

22. Show that for any positive numbers a and b, $(ab)^{\frac{1}{2}} = a^{\frac{1}{2}}b^{\frac{1}{2}}$.

23. Simplify $\left(\left(16x^3\right)^{\frac{1}{4}}\right)^2$.

24. Write the radical expression $\sqrt[7]{a^5b^2}$ in exponential form.

25. Given that $(2^x)^{\frac{3}{2}} = 16$, solve for x.

26. Simplify $\dfrac{81^{\frac{1}{4}}}{3^{\frac{1}{2}}}$.

27. Write $a^{\frac{5}{6}}$ in radical form.

28. If $(2a)^{\frac{1}{2}} = 6$, determine the value of a.

29. Prove that for any positive number x and positive integers m and n, $\left(x^{\frac{1}{n}}\right)^m = x^{\frac{m}{n}}$.

30. Solve for x in the equation $\dfrac{(2x)^{\frac{3}{2}}}{x^{\frac{1}{2}}} = 8$.